Exercise Sheet 6 (Janurary 29th, 2016).

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Exercise 21. Solve the Poisson equation

$$\begin{aligned} -\Delta u &= 4 \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \delta\Omega \end{aligned}$$

in using finite elements.
The domain Ω is the unit circle in two dimensions, the analytic solution is given by

$$\hat{u} = 1 - x^2 - y^2.$$

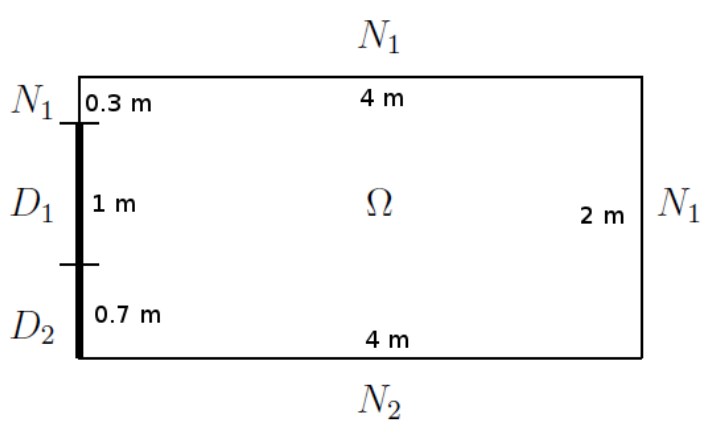
Tasks:

1. Create a uniform mesh of the geometry using DistMesh (available at `http://persson.berkeley.edu/distmesh/` with initial edge length 0.2 cm.
2. Assemble the linear system $Av = f$ using two dimensional linear Lagrangian finite elements and solve it in Matlab.
3. Plot the numerical result and compare the numerical solution to the analytical solution.

Exercise 22. Solve the heat equation

$$\frac{\partial u}{\partial t}(x,t) = D\Delta u(x,t) \quad \text{in } \Omega.$$

The domain Ω represents a coarse model of the heat distribution in a room:



The boundary conditions are given by

$$
\begin{aligned}
u(x,t) &= u_w \quad \text{for } x \in D_1,\ t > 0 \ \ \text{(window)} \\
u(x,t) &= u_h \quad \text{for } x \in D_2,\ t > 0 \ \ \text{(heater)} \\
D\nabla u(x,t) \cdot \nu = 0 & \quad \text{for } x \in N_1 \cup N_2,\ t > 0,\ \text{(perfectly insulated walls)}
\end{aligned}
$$

where ν is the outward normal of the wall.

The parameters of the model are the thermal diffusivity $D = 0.2\ \text{cm}^2/\text{s}$, the outside temperature of an uninsulated window $u_w = 10^\circ$ C, and the temperature of the heater $u_h = 70^\circ$ C.

Tasks:

1. Solve the elliptic steady state problem, where $\partial u/\partial t = 0$.
2. Take the steady state solution as initial condition. Turn off the heating and compute the parabolic problem. How does the temperature change in 2h?