Übungen zu Numerische Methoden II

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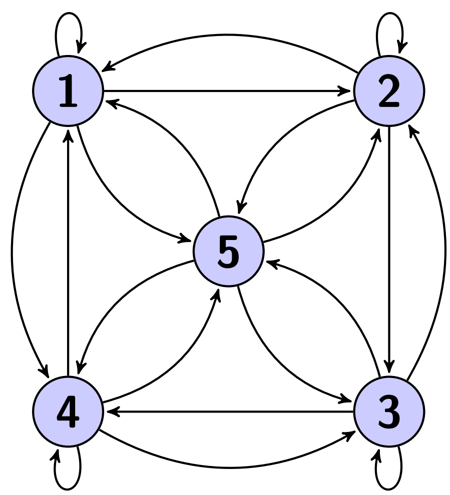
Exercise Sheet 5

1. Consider the following random walk with states $S = \{1, 2, 3, 4, 5\}$. Assume that at each state the transition probabilities to other adjacent states are all equal and the probability to stay at the same state in the next transition is zero.



   (a) Construct the transition probability matrix P.

   (b) Show that the Markov chain of the random walk is irreducible and all the states are recurrent.

   (c) Find the steady-state probability distribution.

2. Consider the transition probability matrix

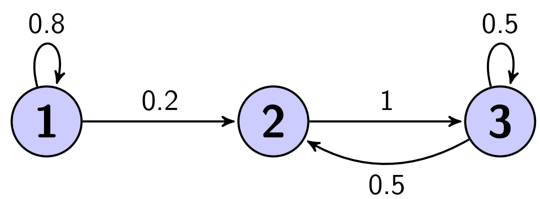
$$P = \begin{pmatrix} 0 & 1 \\ 2/3 & 1/3 \end{pmatrix}.$$

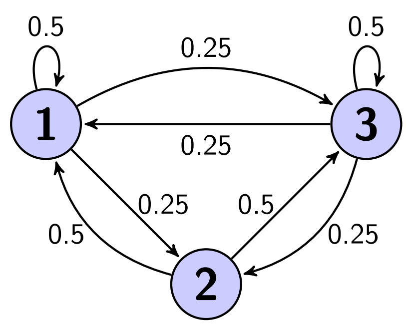
The stationary distribution π is given by

$$\lim_{n\to\infty} P^n X^{(0)} = \pi,$$

for any initial distribution $X^{(0)}$. Compute π using the eigen-decomposition of P.

3. Consider the following two random walks. Construct the transition probability matrices and find the steady-state probability distributions.





4. Consider a game with five levels, where the 5^{th} level is the highest. A player starts at the lowest (1^{st} level) and every time he flips a coin. If it turns up head, the player moves up one level. If tails, he moves down to the 1^{st} level. When the player reaches the highest level, if it turns up heads he stays there and if tails he moves to the lowest level.

   (a) Find the transition probability matrix.

   (b) What is the probability that the player will be in the 3^{rd} level after his second flipping if he started at the 2^{nd} level?

(c) What is the probability that the player will be in the 2^{nd} level after his third flipping for any starting level?

(d) Find the steady-state distribution of the Markov chain (by hand).

5. Consider a Markov chain with transition probability matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Given an initial distribution $\pi(0) = (0, 1, 0, 0)$.

(a) Compute the probability that the state 4 is eventually reached.

(b) Compute the expected time until a recurrent state is entered.