

Exercises for Numerical Methods I

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Sheet 6

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1. Let X, Y be two discrete random variables. The expectation $E(X)$, the variance $\text{Var}(X)$ of X and the covariance $\text{Cov}(X, Y)$ can be expressed by

$$E(X) = \sum_x xP(X = x) := \mu_X,$$
$$\text{Var}(X) = E((X - \mu_X)^2),$$
$$\text{Cov}(X, Y) = E((X - \mu_X)(Y - \mu_Y)).$$

Given that $E(X + Y) = E(X) + E(Y)$ show that:

- (a) $\text{Var}(aX + b) = a^2 \text{Var}(X)$, a, b constants.
(b) $\text{Cov}(X, Y) = E(XY) - \mu_X \mu_Y$.
2. Estimate the parameter θ using the method of maximum likelihood for a density of the form

$$f(x) = (\theta + 1)x^\theta, \quad x \in [0, 1], \quad \theta > 0$$

and random sample with variables:

$$\frac{1}{5}, \frac{2}{5}, \frac{1}{2}, \frac{7}{10}, \frac{4}{5}, \frac{4}{5}, \frac{9}{10}, \frac{9}{10}.$$

3. The density of the Gamma distribution $G(a, \rho)$ is given by

$$f(x) = \frac{a^\rho}{\Gamma(\rho)} x^{\rho-1} e^{-ax}, \quad \text{for } x > 0 \quad \text{and } a, \rho > 0.$$

Find the maximum likelihood estimator of a for $G(a, 2)$.

4. Let X, Y be two random variables with $\text{Var}(X) > 0$, $\text{Var}(Y) > 0$. Consider a linear function

$$g(X) = aX + b, \quad a, b \text{ parameters.}$$

The function g is called the best linear predictor if g estimates Y by minimizing the expectation

$$E((Y - g(X))^2).$$

Compute the parameters a, b such that the mean square error is minimum.

5. Consider X, Y two 1×200 vectors with random values from the standard uniform distribution on the interval $(0, 1)$. Implement in MATLAB an algorithm with inputs X, Y and output the graph of X, Y and the best linear predictor $g(X)$, as found in exercise 4.
6. Consider a finite population of people with size N , containing exactly K musicians. We choose randomly n people without replacement. For a random variable X , the probability mass function for the hypergeometric distribution is given by

$$P(X = x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

and for the binomial distribution,

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},$$

where p is the probability of success. For $N \rightarrow \infty$, the hypergeometric distribution approaches the binomial distribution. Let $N = 75.000$ and $K = 500$. We choose 25 people randomly. Implement in MATLAB an algorithm to compute the probabilities, using both distributions, that

- (a) at most one musician is selected.
- (b) two or three musicians are selected.